Elastic Deflection as an Estimator of Viscous Deformation of Fused Quartz

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Abstract

Room temperature elastic deflection of quartz structures can be used to predict the sag in the structure at high temperature. The difference between elastic and plastic deflection is compared and used to show that for many structures viscous sag is proportional to elastic deformation.

Key Words: Quartz Sag, Quartz Deflection, Plastic, Elastic, Viscosity, Young's Modulus, High Temperature

When planning the use of fused quartz at high temperature, it is of interest to know whether the quartz structure will or will not deform during use. An exact determination of the potential time dependent deformation of the structure can be done by either actual service exposure or by detailed analytical effort. Another approach can be taken to determine the high-temperature deformation of the structure. By comparing the room temperature elastic deflections of the structure and a simple, reference geometry, it is possible to calculate the expected deformation rate for the structure. This paper describes the technique and the rationale for the technique of estimation the sag rate at a function of deflection measurements that can be done at room temperature.

Fused quartz is the simplest silica glass. Its room temperature mechanical behavior is purely linear-elastic, and linear mechanical analysis will determine the magnitude and location of the maximum stresses.

The high-temperature behavior of fused quartz is determined by its state of stress and the viscosity at the temperature. Shear forces cause atoms and molecular groups to move relative to one another, and viscosity is a measure of a liquid's ability to resist shear motion. Viscosity is a liquid state property and it is the reciprocal of the fluidity (h=1/fludidty). Typical units for viscosity are Pascal * seconds and poise (1 Pa*s=10 poise). The viscosity of a glass is a monotonic function of temperature and ranges from greater than 1020 poise (essentially solid) at room temperature to 104 at the working temperature of the glass. A material that has viscosity of 104 poise is stiff, but can be pushed, poked, and pulled into various shapes. Viscosity obeys Arrhenius type of rate equation,i.e., H=hoexp (DG/RT)

Where h is the viscosity, ho is a pre-exponential constant, DG the "the activation energy" or temperature dependence of the viscosity, R the gas constant, and T the absolute temperature.

When a shear stress acts on an elastic structure or body, the body undergoes an almost instantaneous (the time being related to the speed of sound in the material) elastic deflection to a new equilibrium shape. Once the new elastic equilibrium shape has been obtained, any continuing time rate of deformation is linearly related to the shear stress and inversely related to the viscosity. When a shear stress acts on a viscous body, the body starts deforming or flowing with time at a rate that is inversely proportional to its viscosity. Below 800 C, fused quartz is so viscous as to be essentially solid, but above 800 C, deformation by viscous flow is possible.

The solution to many viscosity problems can be found by obtaining the solution to an analogous elastic problem. This is true for highly viscous bodies, such as quartz in the range of use temperature dependence of the viscosity will allow prediction of the high t-temperature deformation of the glass structure.

For linear visco-elastic system such as fused quartz, the ration of viscous deformation rate of a structure to the elastic deflection of that structure is a constant (K) that is the dependent only upon temperature. If K is the ration of deformation rate to elastic deflection, then the ratio can be expressed as:

$$K = \frac{1}{3}\frac{E}{\eta}$$

As an example, let us consider the case of single-point loaded, round rod of radius 'a', with a load span of 'L', and a load of 'P', the deformation rate can be written as:

$$\frac{\text{Deformation}}{\text{Time}} = \frac{2PL^3}{72\pi\eta a^4}$$

The elastic deflection can be written as: Substituting the above relationship for the moment of inertia in the

$$\text{Deflection} = \frac{PL^3}{48EI}$$

Where 'I' is the moment of inertia. For a round rod, the moment of inertia is:

$$I = \frac{\pi(2a)^4}{64}$$

Substituting the above relationship for the moment of inertia into the deflection equation and taking the ratio of the deformation to the elastic deflection we have:

$$\frac{\dfrac{\text{Deformation}}{\text{Time}}}{\text{Deflection}} = \frac{\dfrac{2PL^3}{72\pi\eta a^4}}{\dfrac{PL^3}{48EI}} = \frac{1}{3}\frac{E}{\eta}$$

A similar case can be made for 4-point loading of a round rod. If 'L' is the support length and 'x' is the distance between the load points, then the ratio of the deformation rate to the elastic deflection is:

$$\frac{\dfrac{\text{Deformation}}{\text{Time}}}{\text{Deflection}} = \frac{\dfrac{P(L\text{-}x)(2L^2 + 2Lx\text{-}x^2)}{72\pi\eta a^4}}{\dfrac{Pc(3L2 - 4c)}{48EI}} = \frac{1}{3}\frac{E}{\eta}$$

Where I is the moment of inertia from above and

$$C = \frac{(L - x)}{2}$$

A very important point can be drawn from the above discussion. The ratio of elastic deflection of two structures is also the ratio of their deformation rates. If one has a complicated structure for which it is difficult to calculate the moment of inertia, it is still possible to estimate the in-service deformation rate of that structure. The deformation rate estimation is done by measuring the structure's room temperature elastic deflection, and then multiplying that deflection times the deformation rate/deflection ratio for a similarly loaded simple structure at the temperature of interest. An equation that describes the calculation is given below:

$$\text{Structure Deformation Rate} = \frac{\text{Structure Elastic Deflection}}{\text{Simple Shape Deflection Rate}} \times \text{Simple Shape Deformation Rate}$$

A calculator of deflection and deformation after a given time for simple structures at various temperatures is provided in the attached calculator. In the above equation, either deformation rate or total deformation after a given time can be used:

$$\text{Structure Total Deformation} = \frac{\text{Structure Elastic Deflection}}{\text{Simple Shape Elastic Deflection}} \times \text{Simple Shape Total Deformation}$$

If we have a 1 meter structure that is supported at its ends and carries a uniformly distributed 1000 gram load, it is of interest to estimate its deflection after 1000 hours of exposure at 1000 C. To do the estimation, we measure the room temperature deflection of the structure when it is loaded. We find that elastic deflection to .002cm and the 1000 C, 1000-hour sag to be 0.9746 cm. To estimate the 1000 C, 1000-hour sag deflection of our structure, we use the above equation for total deformation:

$$Sag = \frac{0.002}{0.0713} \times .9746$$

In 1000 hours, at 1000 C, sag of the structure will be 0.0273 cm.